Semidiurnal signals in UT1/LOD due to the influence of tidal gravitation on the triaxial structure of the Earth

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Introduction

The problem

The axial component of Earth rotation, which is expressed by Universal Time UT1, contains small variations due to the influence of tidal gravitation on the triaxial structure of the Earth.

- Predicted quite early by, e.g., Tisserand (1891), Kinoshita (1977), but neglected in computation due to very small size (maximum peak-to-peak variation about 0.1 mas)
- More complete descriptions have been published when the effect became observationally significant
  - Chao et al., 1991, *Geophys. Res. Letters*, Vol. 18, pp. 2007–2010;
  - Wünsch, 1991, *Astron. Nachrichten*, Vol. 312, pp. 321–325;
  - Brzeziński and Capitaine, 2002, *Proc. Journees 2001*, 51–58.
- However, the model is still not included in the IERS Conventions, in contrast to the corresponding effect in polar motion (IERS Conventions 2003, Table 5.1)

Our purpose

- Derive an analytical solution for the lunisolar perturbations in UT1 associated with the triaxiality of the Earth, assuming 2-layer model consisting of elastic mantle and liquid core
- Estimate coefficients of the model and compare them to the available alternative solutions as well as to the ocean tide influences

Lunisolar torque on the triaxial Earth

The torque exerted on the Earth by the external point mass M_d (the Moon or the Sun) at the geocentric position $\vec{r}_d = (r_d \sin\theta_d \cos\lambda_d, r_d \sin\theta_d \sin\lambda_d, r_d \cos\theta_d)^T$, can be evaluated as the negative of the torque on M_d due to the Earth, which yields the following expression

$$\vec{L} = -M_d \, \vec{r}_d \times \nabla U(\vec{r}_d), \tag{1}$$

where $\times$ – vector product, ∇ – gradient operator, and U denotes gravitational potential of the Earth which can be conveniently expanded in spherical harmonics

$$U(\vec{r}) = \frac{GM}{r} \sum_{\ell=0}^{\infty} \sum_{j=0}^{\ell} \left(\frac{R_e}{r}\right)^{\ell} P_{\ell j}(\cos\theta)[C_{\ell j}\cos j\lambda + S_{\ell j}\sin j\lambda], \tag{2}$$

with

G – gravitational constant,

M, R_e – mass and mean equatorial radius of the Earth,

$P_{\ell j}$ – associated Legendre function of degree/order (ℓ, j),

$C_{\ell j}, S_{\ell j}$ – Stokes (un-normalized) coefficients evaluated at $r = R_e$.

Substitution yields the following expression for the axial torque

$$L_3 = -M_d \frac{\partial U(r_d, \theta_d, \lambda_d)}{\partial \lambda_d}. \tag{3}$$

For further derivations it is necessary to know the time dependence of r_d, θ_d, λ_d.

Lunisolar torque on the triaxial Earth

We apply the procedure developed by Woolard (1953) and described by McClure (1974). The time dependence of r_d, θ_d, λ_d is derived from the analytical development the tide generating potential $u(\vec{r})$

$$u(\vec{r}) = \frac{GM_d}{c_d} \sum_{\ell=2}^{\infty} \sum_{j=0}^{\ell} \left(\frac{r}{c_d}\right)^{\ell} P_{\ell j}(\cos\theta) \sum_{s} A_{\ell js} \cos\left[\omega_{\ell js} t + \beta_{\ell js} + j\lambda + (\ell - j)\frac{\pi}{2}\right], \qquad (4)$$

where

c_d – mean distance of the disturbing body,
$\omega_{\ell js}$, $\beta_{\ell js}$ – frequency and phase of the tidal component of degree ℓ, order j and number s,
$A_{\ell js}$ – amplitude of the expansion.

Note: $\omega_{\ell js} \approx j\Omega$, where Ω – angular frequency of diurnal sidereal rotation. Therefore the component $u_{\ell j}$ of the expansion (4) expresses the long periodic (j =0), diurnal (j =1), semidiurnal (j =2), terdiurnal (j =3), etc., tide.

For each term $U_{\ell j}$, $\ell \geq 2$, of geopotential (2) we could derive explicit expressions for both the equatorial and axial torques involving certain components $u_{\ell' j'}$ of tidal potential (4). These expressions take form of harmonic expansion with summation with respect to s. The following table gives review of all components of the lunisolar excitation having influence on Earth rotation, which exceed the level 0.1 microarcsecond (μas).

Lunisolar perturbations in Earth rotation

Table 1: Lunisolar perturbations in Earth rotation. Last column shows the sum of the absolute values of all amplitudes greater than 0.01 μas. Table includes only those components of perturbation for which the total effect exceeds the level of 0.1 μas. Note: UT1 is expressed in angular units using the correspondence 1μs – 15μas.

<table>
<tr><th>Geo-
potential</th><th>Tidal
potential</th><th>Nutation</th><th>Polar motion</th><th>UT1</th><th>Maximum total
amplitude (in μas)</th></tr>
<tr><td>$U_{\ell 0}$
for $\ell = 2, 3, \dots$</td><td>$u_{\ell 1}$</td><td>long
periodic</td><td>retrograde
diurnal</td><td>—
—</td><td>nutation $>10^7$
$+$ precession</td></tr>
<tr><td>U_{31}</td><td>u_{30}</td><td rowspan="2">prograde
diurnal</td><td rowspan="2">long
periodic</td><td>—</td><td>91.3</td></tr>
<tr><td>U_{41}</td><td>u_{40}</td><td>—</td><td>1.0 $+$ drift 5.7 μas/yr</td></tr>
<tr><td>U_{22}</td><td>u_{21}</td><td rowspan="2">prograde
semidiurnal</td><td rowspan="2">prograde
diurnal</td><td>—</td><td>51.6</td></tr>
<tr><td>U_{32}</td><td>u_{31}</td><td>—</td><td>0.2</td></tr>
<tr><td>U_{33}</td><td>u_{32}</td><td>prograde terdiurnal</td><td>prograde semidiurnal</td><td>—</td><td>0.1</td></tr>
<tr><td>U_{31}</td><td>u_{32}</td><td>retrograde diurnal</td><td>retrograde semidiurnal</td><td>—</td><td>0.8</td></tr>
<tr><td>U_{32}</td><td>u_{33}</td><td>retrograde semidiurnal</td><td>retrograde terdiurnal</td><td>—</td><td>0.1</td></tr>
<tr><td>U_{22}</td><td>u_{22}</td><td>—</td><td>—</td><td rowspan="2">semi-
diurnal</td><td>62.9</td></tr>
<tr><td>U_{32}</td><td>u_{32}</td><td>—</td><td>—</td><td>0.3</td></tr>
<tr><td>U_{31}</td><td>u_{31}</td><td>—</td><td>—</td><td>diurnal</td><td>1.1</td></tr>
<tr><td>U_{33}</td><td>u_{33}</td><td>—</td><td>—</td><td>terdiurnal</td><td>0.3</td></tr>
</table>

UT1 variation due to the triaxial figure of the Earth

Consider now the special case of U_{22}. We find the following expression for the axial torque

$$L_3 = -24\,\frac{GMM_dR_e^2}{c_d^3}\,D_{22}\sum_s A_{22s}\sin\left(\omega_{22s}t+\beta_{22s}+2\lambda_{22}\right), \tag{5}$$

in which

$C_{22}+iS_{22} = D_{22}(\cos 2\lambda_{22}+i\sin 2\lambda_{22}) = D_{22}e^{i2\lambda_{22}}$, with $i=\sqrt{-1}$.

Note: if A<B<C are the principal moments of inertia of the Earth, then

- $D_{22} = (B-A)/4MR_e^2$ is proportional to the equatorial dynamical flattening
- λ_{22} is the angle between the x-axis of the TRS and the A-principal axis of inertia

Remarks:

1. The torque is proportional to the equatorial flattening of the Earth. Parameters D_{22}, λ_{22} can be derived with high precision from the satellite determinations of gravity potential.
2. The summation extends over the set of the second-degree semidiurnal tides with coefficients A_{22s}, ω_{22s}, β_{22s} which are known from the tidal developments.
3. The physical mechanism of the effect is shown schematically in the following figure.

UT1 variation due to the triaxial figure of the Earth

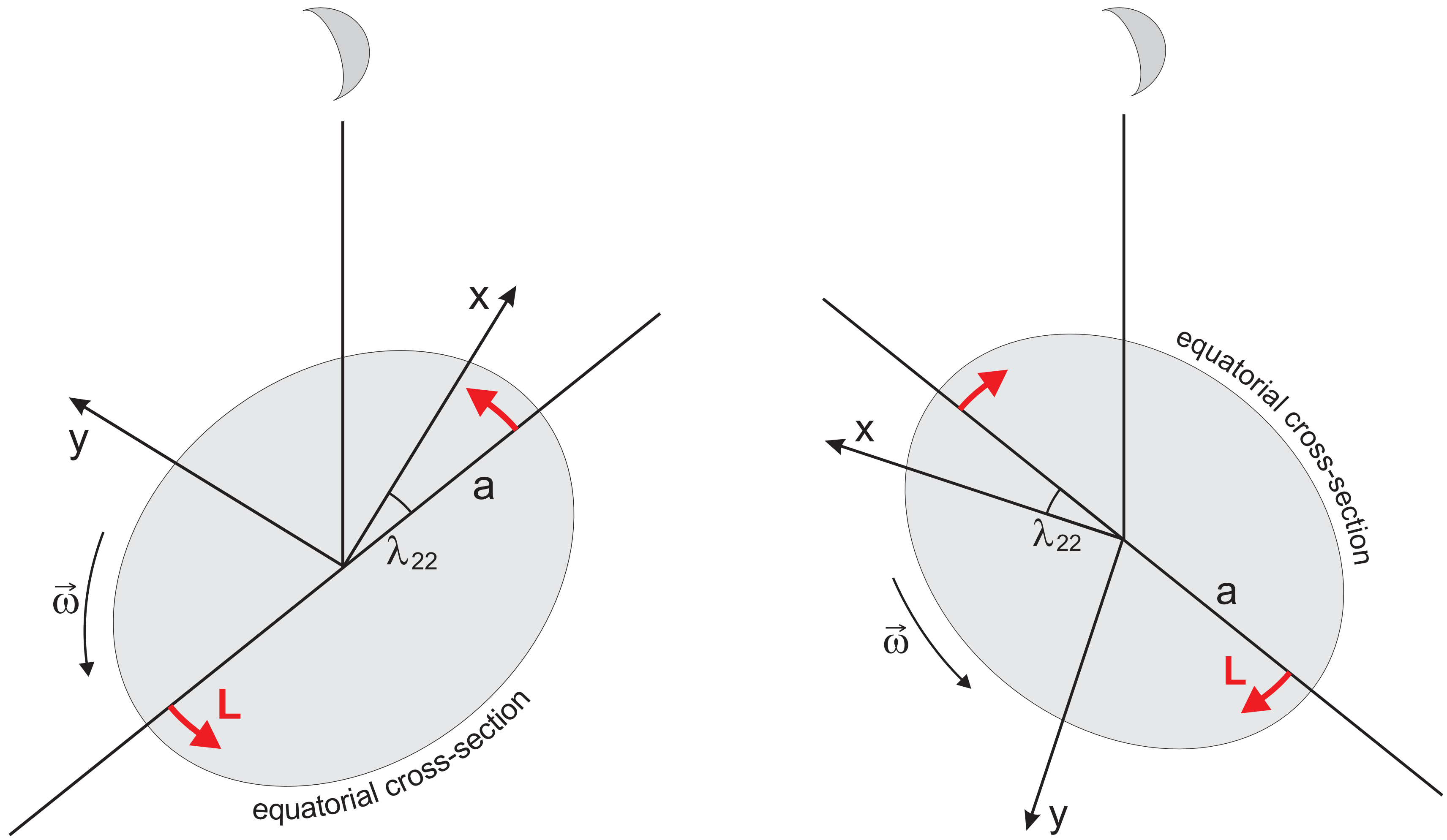

Figure 1: Physical mechanism of the semidiurnal UT1 variation due to the lunisolar tidal torque on the triaxial Earth. Earth rotation is accelerated when the equatorial major semiaxis approaches the line Earth–Moon (left), and decelerated after crossing this line (right). This configuration repeats after 180° corresponding to 12 hours.

UT1 variation due to the triaxial figure of the Earth

Now we estimate the Earth response to the axial torque under the following assumptions:

- The liquid core does not participate in the rotational variations;
- The satellite-observed triaxiality of the Earth comes from the mantle alone;
- Contributions to U_{22} from the elastic deformations of the mantle due to the influence of the u_{22} tidal potential are neglected as a secondary effect.

The solution is

$$\Delta\mathrm{UT1} = \frac{E\,\kappa}{C_m\,\Omega}\iint L_3 dt^2, \tag{6}$$

where

$E = \left(1 + \frac{4}{3}\frac{k_2}{k_s}\frac{C{-}A}{C}\right)^{-1} \approx 0.999$ expresses influence of elastic rotational deformations,

$\kappa = 0.997270$ is the conversion factor between the sidereal and solar time scales,

C_m denotes principal axial moment of inertia of the mantle.

Substitution of the axial torque L_3 expressed by eq.(5) and double integration yields

$$\Delta\mathrm{UT1} = 24\,\frac{E\,\kappa\,GM\;M_d R_e^2}{c_d^3 C_m\,\Omega^3}\,D_{22}\sum_s \frac{A_{22s}\sin\left(\omega_{22s}t+\beta_{22s}+2\lambda_{22}\right)}{\left(\frac{\omega_{22s}}{\Omega}\right)^2}. \tag{7}$$

By inserting $E=1$ and replacing C_m by C we get the corresponding solution for the rigid Earth.

Computational details and results

Parameters of the model

$D_{22} = 1.815528 \times 10^{-6}$, $2\lambda_{22} = -29.8581^\circ$ from the geopotential expansion JGM-3

Coefficients of the TGP: Hartmann and Wenzel (1995) (after conversion to present format)

$C/C_m = 1.1283$ from model PREM

Argument convention

Each tidal argument of expansion (7) is expressed below using the Woolard convention

$$\omega_{22s}t + \beta_{22s} + 2\lambda_{22} = 2(\chi + \lambda_{22}) + k_1 l_m + k_2 l_s + k_3 F + k_4 D + k_5 \hat{\Omega}, \qquad (8)$$

where χ =GMST+π and l_m, l_s, F, D, $\hat{\Omega}$ are the so-called "fundamental arguments" and $k_1, \ldots, k_6$ are integer coefficients identifying the tidal component.

Results

Table 2 below shows coefficients of expansion (7) computed for the rigid Earth and for the model comprising the elastic mantle and the liquid core. Our estimates are compared to the results of Wünsch (1991) and Chao et al. (1991). We also shown for comparison the amplitudes of the ocean tide influences (IERS Conventions, 2003, Table 8.3b).

UT1 variation due to the triaxial figure of the Earth

Table 2: Semidiurnal variations of UT1 due to the action of the lunisolar tidal potential on the triaxial Earth. The amplitudes for a rigid Earth (denoted "Rigid") and for elastic Earth with liquid core ("2-layer") have been computed using the TGP expansion of Hartmann and Wenzel (1995). Our results are compared to the models of Wünsch (1991) ("Wu91"), Chao et al. (1991) ("Ch91"), and to the ocean tide contributions (OTAM).
Listed are all terms with amplitudes larger than 0.5 μas. Units are μas; Φ denotes GMST$+\pi-14.92905°$.

Argument						Tidal component			UT1					
Φ	l_m	l_s	F	D	$\hat{\Omega}$	Doodson number	Darwin symbol	Origin	Period day	Rigid sin	2-layer sin	Wu91 sin	Ch91 sin	OTAM amp.
2	–2	0	–2	0	–2	235.755	$2N_2$	M	0.5377239	0.73	0.83	0.74	—	10.0
2	0	0	–2	–2	–2	237.555	μ_2	M	0.5363232	0.88	0.99	0.89	—	11.6
2	–1	0	–2	0	–2	245.655	N_2	M	0.5274312	5.34	6.02	5.34	4.8	61.5
2	1	0	–2	–2	–2	247.455	ν_2	M	0.5260835	1.01	1.14	—	—	11.4
2	0	0	–2	0	–1	255.545	M_2'	M	0.5175645	–1.00	–1.13	–1.01	—	9.9
2	0	0	–2	0	–2	255.555	M_2	M	0.5175251	26.84	30.28	26.84	29.0	265.5
2	1	0	–2	0	–2	265.455	L_2	M	0.5079842	–0.73	–0.82	–0.74	—	6.6
2	0	–1	–2	2	–2	272.556	T_2	S	0.5006854	0.68	0.77	0.69	—	6.6
2	0	0	–2	2	–2	273.555	S_2	S	0.5000000	11.65	13.15	11.66	13.0	113.3
2	0	0	0	0	0	275.555	K_2	M	0.4986348	2.15	2.43	2.16	3.6	31.5
2	0	0	0	0	0	275.555	K_2	S	0.4986348	1.00	1.13	0.99		
2	0	0	0	0	–1	275.565	K_2'	M	0.4985982	0.94	1.06	0.95	—	9.5
Sum of all amplitudes >0.01 μas (68 terms)										55.71	62.86			

Conclusions

We considered in this work the lunisolar perturbations in UT1 associated with departures of the Earth's dynamical figure from the rotational symmetry. We arrived to the following conclusions:

- The only terms with amplitudes exceeding the truncation level of 0.5 μas are 11 semidiurnal harmonics due to the influence of the second degree tidal potential u_{22} on the equatorial flattening of the Earth expressed by the Stokes coefficients C_{22}, S_{22}.
- There is an excellent agreement between the amplitudes derived by Wünsch (1991) and our estimates assuming model of the rigid Earth. The only important difference is the term ν_2 which seems to be overlooked in the development of Wünsch.
- Accounting for the presence of the liquid core increases all amplitudes by the factor $\mathrm{C}/\mathrm{C}_m =$ 1.128. Our amplitudes computed for an elastic Earth with liquid core are in reasonable agreement with those derived by Chao et al. (1991), but the last model was not complete.
- Estimated effect is superimposed on the ocean tide influence having the same frequencies but 9 to 11 times larger amplitudes. Nevertheless, its maximum peak-to-peak size is about 7 μs (105 μas) hence definitely above the current uncertainty of UT1.
- We recommend adding this model to the set of procedures provided by the IERS Conventions. Comparison with the model of prograde diurnal polar motion due to the triaxiality (IERS Conventions, Tab. 5.1) shows that: 1) the two effects are of similar size, 2) there is consistency between the underlying dynamical models, parameters employed, etc.